

Star formation efficiency in turbulent clouds

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ABSTRACT

Here we present a simple, but nevertheless, instructive model for the star formation efficiency (SFE) in turbulent molecular clouds. The model is based on the assumption of log-normal density distribution which reflects the turbulent nature of the interstellar medium (ISM). Together with the number count of cloud cores, which follows a Salpeter-like core mass function (CMF), and the minimum mass for the collapse of individual cloud cores, given by the local Jeans mass (M_J), we are able to derive the SFE for clouds as a function of their Jeans masses. We find a very generic power-law, $SFE \propto N_J^{-0.26}$, where $N_J = M_{\text{cloud}}/M_J$ and a maximum $SFE_{\text{max}} \sim 1/3$ for the Salpeter case. This result is independent of the turbulent Mach number but fairly sensitive to variations of the CMF.

Key words. ISM: clouds – ISM: structure – ISM: kinematics and dynamics – Turbulence

1. Introduction

Molecular clouds, the birthplaces of stars in galaxies, are pervaded by turbulent motions, which to a large extent determine the cloud's density distribution (see e.g. reviews by Mac Low & Klessen 2004; Ballesteros-Paredes et al. 2007; Dobbs et al. 2013; Padoan et al. 2013, and references herein). The distribution function of those density fluctuations is commonly described by a log-normal distribution (e.g., Vazquez-Semadeni 1994; Padoan & Nordlund 2002; Federrath et al. 2008). Furthermore, the mass distribution of cores and clumps¹, i.e. the core mass function (CMF), within the molecular cloud seems to follow a power-law distribution, similar to the stellar initial mass function (IMF) (see e.g., Alves et al. 2007; Rathborne et al. 2009; André et al. 2010; Könyves et al. 2010; André et al. 2012). Based on those ingredients there are a number of analytic approaches to calculate the efficiency of turbulent molecular clouds to form stars. In particular, Padoan (1995) used a procedure combining the distribution of cores according to the turbulent property of the parent cloud with the mass distribution of those cores. We comment again on the Padoan (1995) approach later in this letter. Krumholz & McKee (2005) derived an efficiency of star formation per free-fall time by considering the fraction of mass which exceeds a critical density determined by the condition of gravitational instability without taking the CMF into account, because they are only interested in the rate at which stars form. Similar approaches, i.e. considering the mass fraction of the density-PDF that exceeds a critical density, were also used by e.g. Padoan & Nordlund (2011); Hennebelle & Chabrier (2011); Kainulainen et al. (2014), to derive a star formation efficiency.

In this letter, we would argue that it is not sufficient to identify the mass of the high density fluctuations to calculate the star formation efficiency, because, first this gives only a *lower* mass-limit and second those high-density cores are embedded in lower mean-density clumps which might still be able to collapse. Therefore, we derive an upper limit for the star formation efficiency by explicitly calculating the highest-mass core within

the entire cloud which is able to collapse by gravitational instability.

2. Model description

We start with the canonical form of the probability distribution function (PDF) of density fluctuations in a turbulent cloud,

$$p(s) = \frac{1}{\sqrt{2\pi}\sigma^2} \exp\left(-\frac{(s-s_0)^2}{2\sigma^2}\right) \quad (1)$$

with $s = \ln(\rho/\rho_0)$ and $s_0 = -1/2 \sigma^2$. The variance of this PDF depends on the turbulence as $\sigma^2 = \ln(1 + b^2 \mathcal{M}^2)$, where for simplicity we neglect the impact of magnetic fields (see e.g., Vazquez-Semadeni 1994; Padoan & Nordlund 2002; Federrath et al. 2008). The parameter b is related to the type of turbulence (see again Federrath et al. 2008) but does not play a crucial role in our consideration as we will see later on.

Obviously only those cloud cores will collapse and form stars which exceed the Jeans mass

$$M_J \approx \left(\frac{c_s}{\sqrt{G}}\right)^3 \frac{1}{\sqrt{\rho}} \propto \rho^{-1/2} \quad (2)$$

at their mean density ρ and temperature $T \propto c_s^2$ (c_s is the speed of sound and G the gravitational constant).

Using the mass of the cloud that exceeds the density ρ

$$\begin{aligned} M(s) &= M_{\text{cloud}} \int_s^\infty ds p(s) \\ &= \frac{M_{\text{cloud}}}{2} \left[1 - \text{erf}\left(-\frac{s-s_0}{\sqrt{2}\sigma}\right) \right] \end{aligned} \quad (3)$$

we can calculate the minimum mass M_{min} of a turbulent cloud which is Jeans unstable, i.e.

$$M_{\text{min}} : M(s) = M_J(s). \quad (4)$$

Given the fact that the cloud is fragmented by the same nature of turbulence, the mass at a given density is not located in

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¹ We use terms cores and clumps interchangeably for connected sub-regions within the considered cloud.

single cloud cores but rather distributed according to a *core mass function* (CMF). Interestingly, the CMF has a very similar shape the *stellar* initial mass function (IMF) and is often assumed to follow a Salpeter distribution, i.e.

$$\text{CMF} \equiv \frac{dN}{d \ln M} \propto M^{-\alpha} \quad (5)$$

with $\alpha \approx 1.35$ (Alves et al. 2007). Now the key point is the normalisation of this number distribution which differs from cloud to cloud. The normalisation of the CMF is given by the total mass of the cloud:

$$M_{\text{cloud}} = C \int_{M_{\text{low}}}^{M_{\text{cloud}}} dM M^{-\alpha}. \quad (6)$$

It follows that ($\alpha \neq 1$)

$$M_{\text{cloud}} = C \frac{M_{\text{low}}^{-\alpha+1}}{\alpha - 1} \quad (7)$$

assuming that $M_{\text{cloud}} \gg M_{\text{low}}$. Unfortunately on first sight, this result depends strongly on the mass of the cores, M_{low} which still contribute to the (Salpeter) distribution. But fortunately, this mass can easily be determined as we expect that the CMF is largely governed by the impact of self-gravitating cloud cores (e.g., Kainulainen et al. 2011; Kainulainen & Tan 2013; Girichidis et al. 2014)². In this case the lowest-mass core which gives rise to the CMF is given by M_{min} , i.e. the core which is just Jeans unstable. Now the number distribution of the cores within the cloud which masses are larger than M is given by

$$N(M) = C \int_M^{M_{\text{cloud}}} dM M^{-\alpha-1} \quad (8)$$

with $C = (\alpha - 1) (M_{\text{cloud}}/M_{\text{min}}) M_{\text{min}}^\alpha$ from Eq. (7). Hence,

$$N(M) = \frac{\alpha - 1}{\alpha} \left(\frac{M_{\text{cloud}}}{M_{\text{min}}} \right) \left[\left(\frac{M}{M_{\text{min}}} \right)^{-\alpha} - \left(\frac{M_{\text{cloud}}}{M_{\text{min}}} \right)^{-\alpha} \right] \quad (9)$$

Now we can search for the largest (locally connected) cloud core which is found by the condition

$$M_{\text{thres}} : N(M_{\text{thres}}) = 1. \quad (10)$$

This condition tells us that there is only *one* core with mass M_{thres} , whereas cores that exceed this mass do *not* exist in the cloud, i.e. $N(M) < 1$. Hence, cores with $M_{\text{core}} > M_{\text{thres}}$ can *not* contribute to star formation, solely because they are *not present*. Otherwise, cores that are smaller than M_{thres} become increasingly more abundant for decreasing core masses (as long as $\alpha > -1$). That means one could, in principle, determine the smallest cores that might contribute to star formation by $M(s)/N(s) > M_J(s)$. But this conditions does ignore that high density cores (which are the ones with the smallest masses) are embedded in larger, more massive cores which are able to form stars (Vazquez-Semadeni 1994). Therefore, this condition does not apply for our consideration of the SFE.

At this point we briefly have to comment on a similar approach discussed by Padoan (1995), (see also Padoan & Nordlund 2002). Here the efficiency to form stars is assumed to be essentially $M(m) \times N(M)$ (see Eqs. (21) and (24) of Padoan 1995), i.e. by the total mass M of *all* clumps with mass m in the *entire* cloud *times* the frequency of those clumps in the cloud. Hence,

² This is particular plausible if we assume that the IMF and CMF have a similar origin.

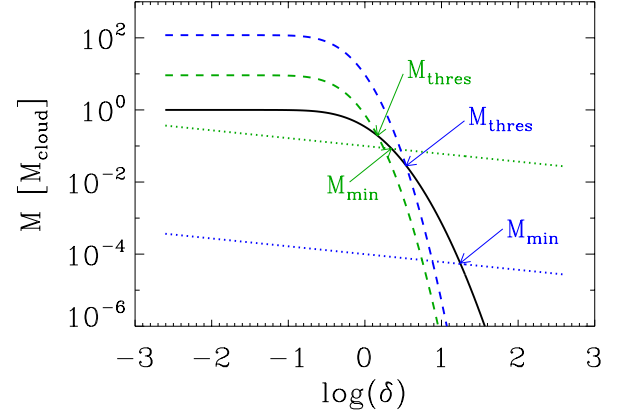


Fig. 1. Shows the concept of the presented model on the SFE and density threshold for star formation. The intersection of the Jeans mass (dotted lines) with the mass distribution of the cloud (solid line) gives us the smallest cores (by mass) which still are able to collapse. This minimum mass determines the upper end of the CMF. Calculating the individual core masses within the cloud (dashed lines) we can determine the largest core (by mass) which is present in the cloud (intersections with the solid line). Its mass is given by M_{thres} (see Eq. 10) and the ratio $M_{\text{thres}}/M_{\text{cloud}}$ gives us the upper limit for the SFE. Here we show two examples with $N_J = 10$ (green lines) and $N_J = 10^4$ (blue lines).

the outcome of this convolution does not reflect the total mass of unstable cores.

Now we can calculate an upper limit for the SFE of molecular clouds (MCs) and giant molecular clouds (GMCs) as a function of their number of Jeans masses,

$$\text{SFE} \equiv \frac{M_{\text{thres}}}{M_{\text{cloud}}}, \quad (11)$$

where we use the number of Jeans masses

$$N_J \equiv \frac{M_{\text{cloud}}}{M_J(\rho_0)} \quad (12)$$

to quantify the instability of the cloud.

In Fig. 1 we summarise the concept of our model to calculate the SFE based on two examples of N_J , where we use $\delta \equiv \rho/\rho_0$ for convenience.

3. Results

Together with Eq. (9) and the condition Eq. (11) the star formation efficiency can be expressed as

$$\begin{aligned} \text{SFE} &= \left[\left(\frac{\alpha}{\alpha - 1} \right) \left(\frac{M_{\text{cloud}}}{M_{\text{min}}} \right)^{-1} + \left(\frac{M_{\text{cloud}}}{M_{\text{min}}} \right)^{-\alpha} \right]^{-1/\alpha} \left(\frac{M_{\text{cloud}}}{M_{\text{min}}} \right)^{-1} \\ &\approx \left(\frac{\alpha - 1}{\alpha} \right)^{1/\alpha} \left(\frac{M_{\text{cloud}}}{M_{\text{min}}} \right)^{(1-\alpha)/\alpha}, \end{aligned} \quad (13)$$

where we assumed $\alpha > 1$ for the approximation. Measuring the cloud mass in terms of its Jeans mass at the mean density we see from Eq. (13) that

$$\text{SFE} \propto N_J^{(1-\alpha)/\alpha} \quad (14)$$

which results in $\text{SFE} \propto N_J^{-0.26}$ for the Salpeter case.

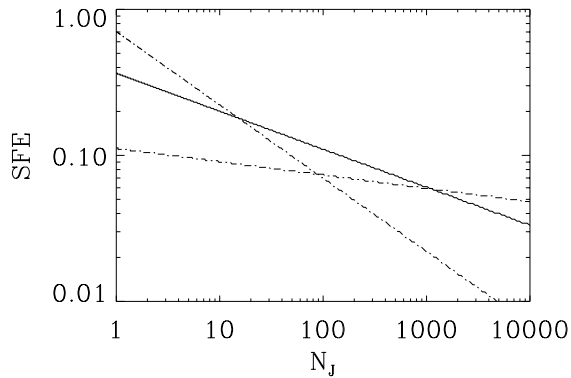


Fig. 2. SFE for our fiducial model, i.e. $M = 1$, $\alpha = 1.35$ (solid line), and for $\alpha = 2$ and $\alpha = 1.1$ (upper and lower dashed line, respectively).

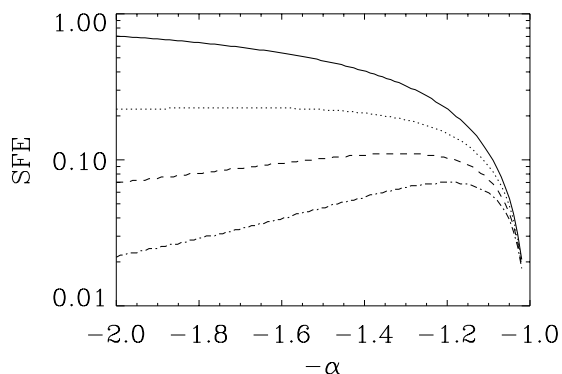


Fig. 3. SFE as a function of the CMF-slope α for different instability parameters. The lines from top to bottom are for $N_J = 1, 10, 100$ and 1000 , respectively. Up to $N_J \lesssim 10$ there is a clear decreasing trend of the SFE with decreasing concentration of the cloud. For massive clouds, this trend is reversed up to a minimal value of $\alpha = \alpha_{\min}(N_J)$ (see text).

Obviously, the definition Eq. (11) is the largest value a cloud could achieve if all the mass of unstable cores are converted instantaneously into stars ignoring all kinds of additional effects like feedback from the stars themselves. But this picture incorporates the effect of reduced accretion onto stars by fragmentation of the cloud, i.e. this model quantifies the consequence of *Fragmentation Induced Starvation* (FIS) (Peters et al. 2010; Girichidis et al. 2012), which can be seen from Fig 2. The more unstable the cloud, quantified by N_J , the less efficient it can form stars because it is more prone to fragmentation with a number of fragments which are not Jeans unstable anymore. Interestingly, in the Salpeter case ($\alpha = 1.35$) the maximal SFE is $\sim 1/3$ ³ for clouds with $M_{\text{cloud}} \approx M_J$. This applies, for instance, to isolated Bok globules like Barnard 68 which might be barely unstable (Alves et al. 2001). Even such low-mass clouds could only convert at most $\sim 1/3$ of their mass into stars, even without any feedback, because they will fragment while they are collapsing.

Also interesting is the fact that the SFE decreases with decreasing concentration of the cloud (decreasing α) for less unstable systems ($N_J \lesssim 10$). Again the reason is the fragmentation property of the cloud. Less concentrated clouds are more susceptible to fragmentation than clouds with a high density con-

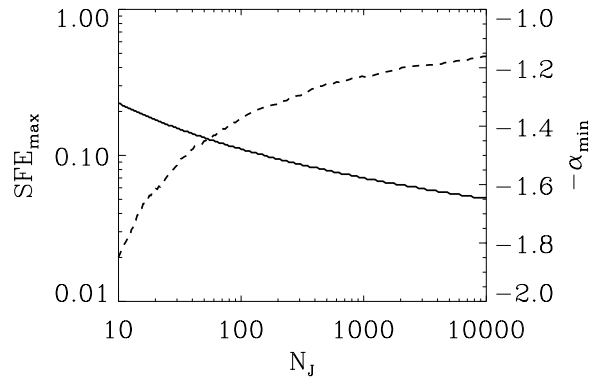


Fig. 4. The maximal star formation efficiency, SFE_{\max} (solid line, left axis) and the appropriate CMF-slope α_{\min} (dashed line, right axis) as a function of the number of Jeans masses, N_J . These values are determined by the fact that for rather unstable clouds the SFE as a function of α reaches a maximum at α_{\min} (see Fig. 3 and text).

centration. This behaviour is intensively studied in Girichidis et al. (2011) and Girichidis et al. (2012) where the collapse of clouds with various density profiles were investigated. Nevertheless, the situations gets a bit more complicated for more unstable clouds ($N_J > 10$) as seen in Fig. 3. Here, less concentration of the CMF helps to *increase* the SFE up to a certain maximal value depending on N_J and α . Hence, for more unstable clouds, the enhanced fragmentation helps to a certain degree to increase the SFE as such fragments are still Jeans unstable and therefore contribute to star formation. This competition between constructive fragmentation and rapid collapse is only efficient up to a minimal concentration, α_{\min} of the cloud (e.g., for $N_J = 1000 \rightarrow \alpha_{\min} \approx 1.2$). For less concentrated clouds, $\alpha \lesssim 1.1$, the SFE becomes essentially independent of N_J and approaches zero in the limiting case $\alpha \rightarrow 1$.

Another interesting aspect which one obtains from the above consideration of the threshold mass, M_{thres} , is the threshold density, ρ_{thres} , for the onset of star formation⁴. Going back to the mass distribution $M(s)$ given by Eq. (3), one can read off ρ_{thres} from the solution of Eq. (10).

We present the result for different values of α in Fig. 5. First of all we see, that there is very little dependence of ρ_{thres} on the instability of the cloud (in the $\alpha = 1.1$ case it becomes almost independent of N_J) and $\rho_{\text{thres}}/\rho_0$ is of order unity. This is not too surprising as we only consider globally unstable clouds in the first place. Again, only for $N_J \lesssim 10$ we find a clear trend of ρ_{thres} with the cloud concentration α : Less concentrated clouds need a larger threshold density to produce stars compared to those with a steeper CMF. For more unstable clouds, $N_J > 10$, the competition between fragmentation and collapse does not lead to such a clear trend with the cloud concentration.

Mach number dependence So far we presented our results for transonic molecular clouds with $M = 1$ (assuming $b = 1$, see also Eq. (1) and below). But it turns out that neither the type of turbulence nor its strength has a large impact on our results. This can already be seen from Eq. (13) which has only a very weak dependence on M_{min} , i.e. on the quantity which depends on M . We tested the Mach number dependence of the SFE nu-

⁴ Actually, the discussion with Marcel Völschow on a self-consistent description of a SF-density threshold spawned this project.

³ the more precise number is 36.6%

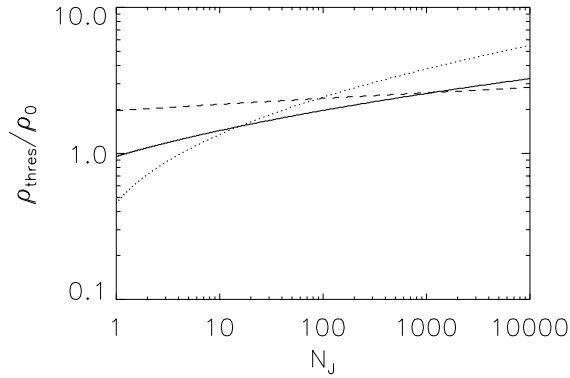


Fig. 5. The threshold density for star formation for $\alpha = 1.35$ (solid line), $\alpha = 1.1$ (dashed line) and $\alpha = 2$ (dotted line). Similar to the SFE, only for $N_J \lesssim 10$ there is a trend of a decreasing threshold with increasing CMF concentration (see also text).

merically using the basic equations, but could not see any visible difference. Hence, we omit a plot showing SFE as a function of \mathcal{M} . The weak dependency of SFE on \mathcal{M} can be understood as follows: Larger Mach numbers result in wider distributions of the density-PDF and therefore would give rise to a larger density threshold (or smaller M_{\min}) for the same Jeans mass. But a wider PDF also reduces the overall instability of the cloud, i.e. reduces N_J . Both effects almost compensate each other. But please note, that already the dependency of M_{\min} on \mathcal{M} is very weak as can be seen from Fig. 1.

4. Conclusions

Here we presented a simple model for the star formation efficiency in turbulent molecular clouds. The model is based on the assumption of log-normal density distribution which reflects the turbulent nature of the ISM. Similar to previous analytic studies, we use this distribution to estimate the minimum mass which can actually collapse by gravitational instability. Any cores that mass exceeds M_{\min} are also Jeans-unstable, but, according to the density distribution, have a lower mean density and are less frequent than lower-mass cores. The latter statement reflects the observed distribution of core masses. But following the CMF, not all low-density regions exist as connected clumps, and hence are not Jeans-unstable. Combining the density-PDF and the CMF we calculate largest core within the cloud which is still able to collapse. This in turn can be used to infer an upper limit for the SFE. For a given slope of the CMF, we find a very generic power-law, $\text{SFE} \propto N_J^{-(\alpha-1)/\alpha}$ and a maximum $\text{SFE}_{\max} \approx 0.37$ for the Salpeter case. Again, this result is independent of the turbulent Mach number.

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